BUREAU OF MANAGEMENT CONSULTING

BUREAU DES CONSEILLERS EN GESTION

REPORT ON THE
LATIN AMERICAN WORLD MODEL

This Model was presented by members of the Fundacion Bariloche at the Second Symposium on Global Modelling sponsored by The International Institute for Applied Systems Analysis.

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1. PREFACE

This report describes the Latin American World Model in general terms and provides a critique of it. The model was developed by members of the Fundacion Bariloche, a non-profit organization located in Argentina, and was presented at the Second Global Modelling Symposium in Baden, Austria, October 1974. This symposium was sponsored by the International Institute for Applied Systems Analysis and included among the participants were systems scientists from twenty-one member countries.

2. INTRODUCTION

Work to develop a Latin American World Model was initiated subsequent to a joint meeting of the Club of Rome and the Institute Universitaro de Pesquisas due Rio de Janiero in 1970. This meeting had been convened to discuss results of the World Model III developed by Meadows at MIT and in the course of the discussions, a number of questions were raised concerning the basic assumptions of the Meadows Model. As a result of this, a decision was made to develop a Latin American World Model and to commend this work to the Fundacion Bariloche. A feasibility study was undertaken with financial support from the Club of Rome and subsequent model development was funded by the International Development Research Center, Ottawa, Canada.

The Latin American World Model constitutes an important step in the evolution of Global Models. Its significance derives from three principal features. Perhaps most striking, is the evident political perspective within which model development has taken place. The Latin American World Model has been constructed in reaction to the basically Malthusian context of which the earlier Meadows model gives evidence. In rejecting Meadows' assumption of world resources dwindling under the pressure of rising population, the Latin American World Model seeks to apply a developing nation's perspective to global modelling. It rejects the notion that physical obstacles pose the main threat to the continued development of mankind and begins instead from the premise that changing value systems and new forms of social organization are the key issues for survival.

A second important feature of the model is that it is normative. Normative models are based on the notion that systems seek to achieve prescribed goals. Accordingly, a system model must incorporate both a concise statement of the goal and a mechanism for directing the model behaviour towards it. This contrasts with the descriptive modelling approach which essentially projects forward from past behaviour.

Adopting the normative approach creates a logical need for the incorporation of assumptions about a value system and social organization into the model. This is a conjunction of circumstances which the Bariloche team have most ably adapted to their purpose.

A third important feature of the Latin American World Model is that the model identifies a small number of key variables and subordinates the model behaviour to these. This is partly a consequence of the normative character of the model but is no doubt also a result of the model being highly structured. This is equivalent to noting that there are relatively fewer linkages between sub-elements of the Latin American World Model than exist between similar elements in other world models.

In what follows, this report will attempt to convey an understanding of how the Bariloche Model works, how the model differs from earlier work by Meadows, some implications of these differences and finally, a critique of the model.

3. MODEL OVERVIEW

To understand the Latin American World Model, it may be useful to view its development in the context of world models generally and in particular to compare its structure with that of the Meadows model.

Meadows work was sponsored by the Club of Rome and constituted an early milestone in the development of World Models. His work is described in "Limits of Growth" published in 1970, which set the stage for wider interest in world modelling. In recent years a number of global models have been developed and the Latin American World Model is one of these, notable perhaps for its departure from the Meadows approach.

Global Models generally work from the premise that five key variables determine the future limits of growth on earth. These are population, natural resources, food production, industrial production and pollution. Interrelationships between these variables are complex in the extreme. Nevertheless, their broad outline can be depicted as in Figure 1 where arrows denote primary cause and effect relationships. Referring to the figure, resource levels influence production rates, which in turn affect pollution and demography. However both pollution and demography have a counter effect on production, as for example when excessive pollution levels force a cutback in production or when labour shortages reduce production levels. At the same time, pollution affects demography.

The multiplicity and circular nature of these primary relationships creates a need for global modelling. The purpose of the model being simply to demonstrate the system behaviour implied by a given set of assumed interrelationships.

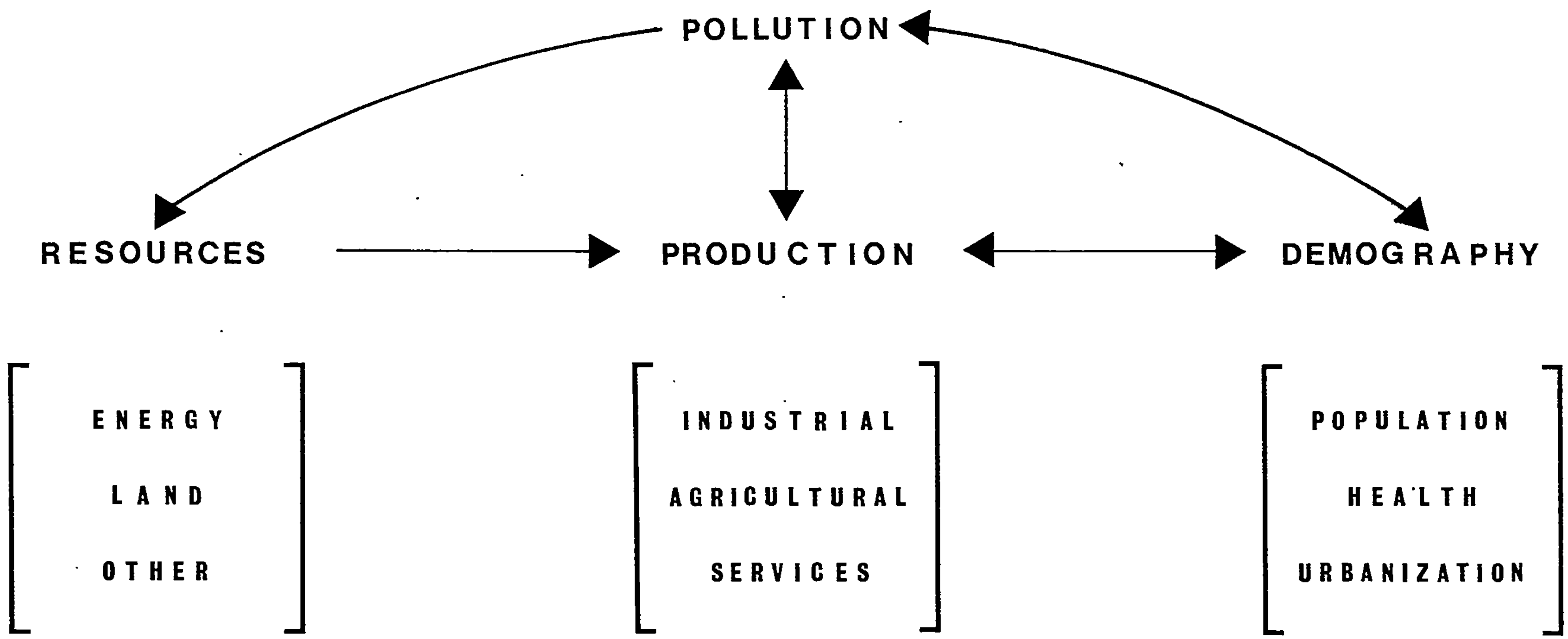

FIGURE 1 : ELEMENTS OF WORLD MODELS

Figures 2 and 3 illustrate the assumptions made about primary relationships in the Meadows and Bariloche* models respectively. The overall structures are similar but an obvious difference lies in the handling of 'pollution' and 'energy and other resources.' In the Bariloche model these are held to pose no serious global threat and are eliminated from the dynamic model. In the Meadows model they comprise a vital part of the dynamic model structure.

As mentioned earlier, the Latin American World Model is normative. This contrasts markedly with the Meadows model which is descriptive. Figures 4 and 5 show the implications of these two approaches in terms of the way in which economic flows of capital and labour are handled in the Meadows and Bariloche models respectively. In the Meadows model, resources are channelled from industrial production into other sectors according to a set of descriptive assumptions that seek to maintain a supply demand balance. In the Bariloche model a goal is established which is to maximize life expectancy. Resources of capital and labour are then channelled from industrial production into other productive sectors according to a prescribed allocation rule that directs the system toward this goal.

As a final point of contrast, the Meadows Model views the world as a single entity whereas the Bariloche Model divides the globe into world areas as follows:**

a) developed countries including, the USA, Russia, Europe, Japan, Canada, New Zealand, Israel, Lebanon;

b) Latin America and the Caribbean;

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* The terms 'Latin American World Model' and 'Bariloche Model' are used interchangeably.

** Countries with a population under one million are excluded from the model.

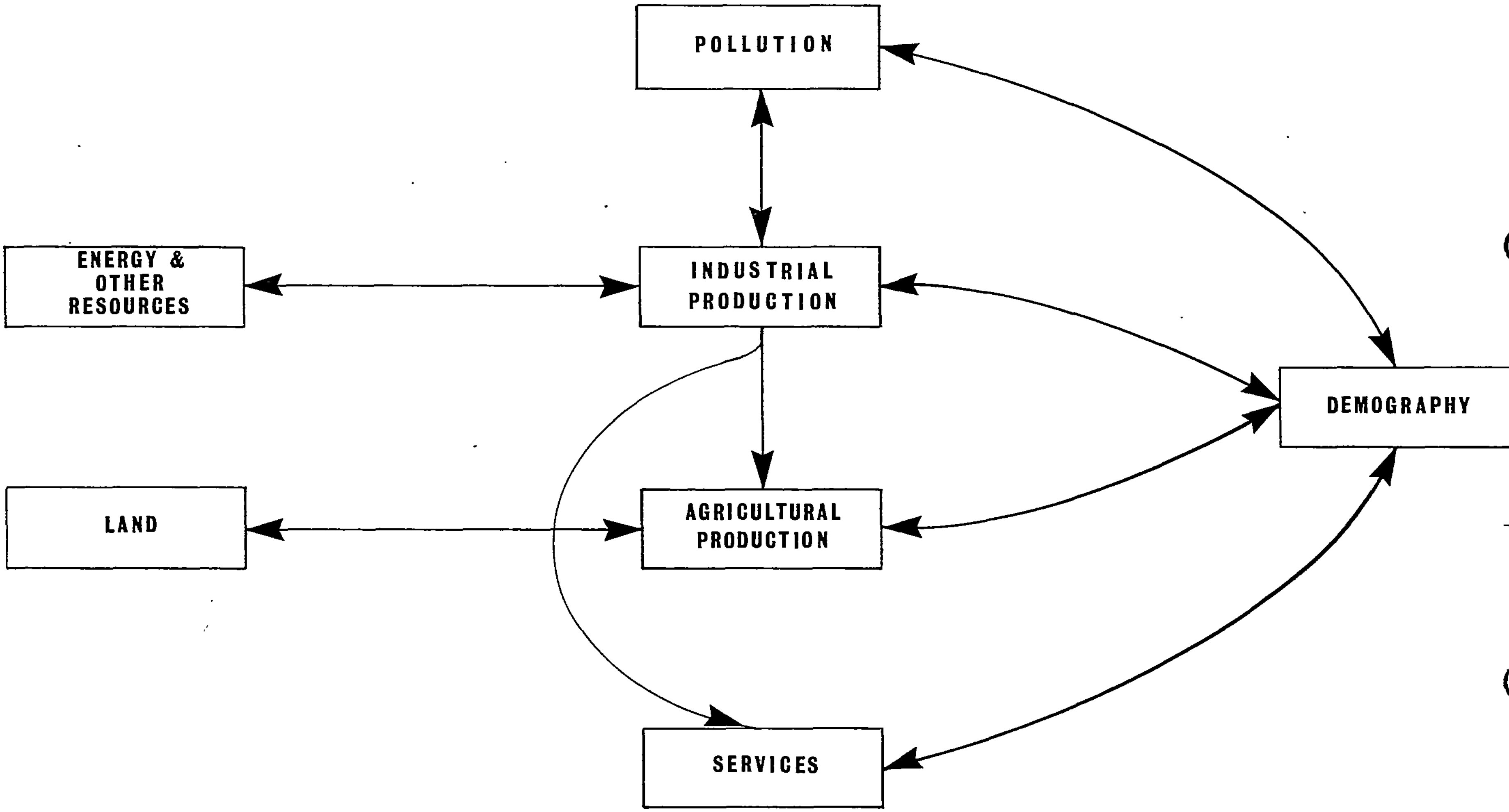

FIGURE 2: OVERVIEW OF MEADOWS MODEL

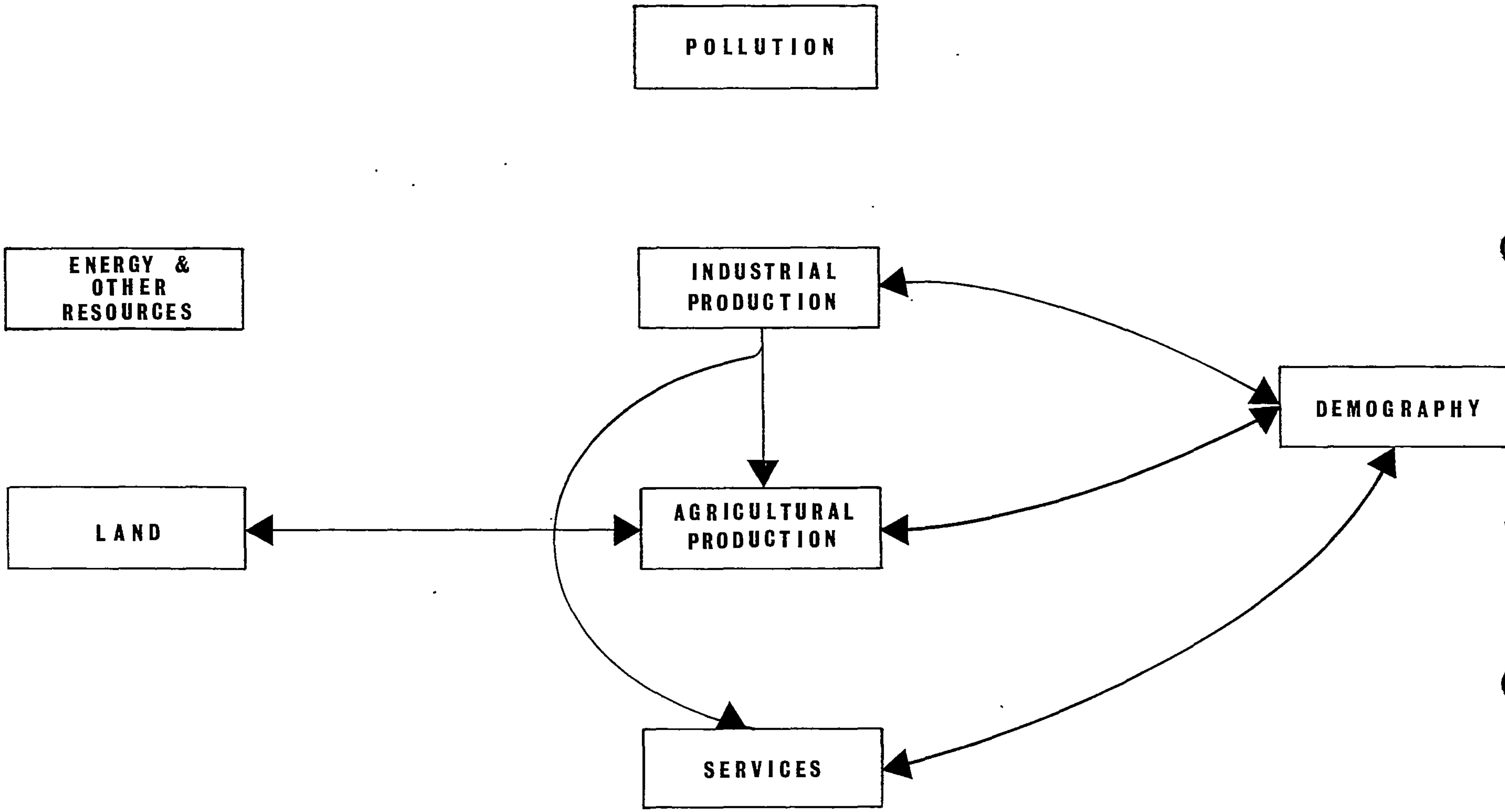

FIGURE 3: OVERVIEW OF BARILOCHE MODEL

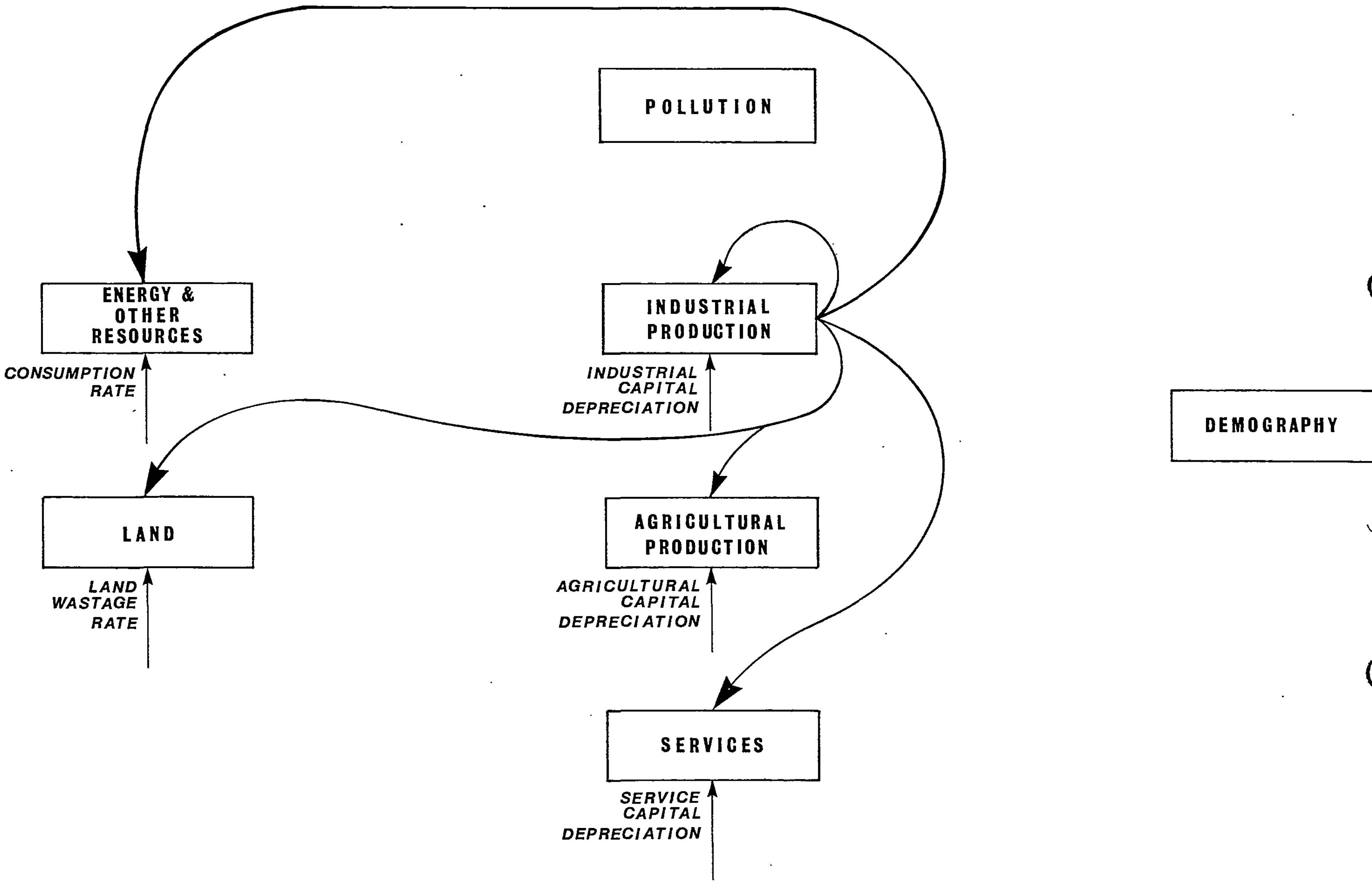

FIGURE 4 : CAPITAL FLOWS IN MEADOWS MODEL

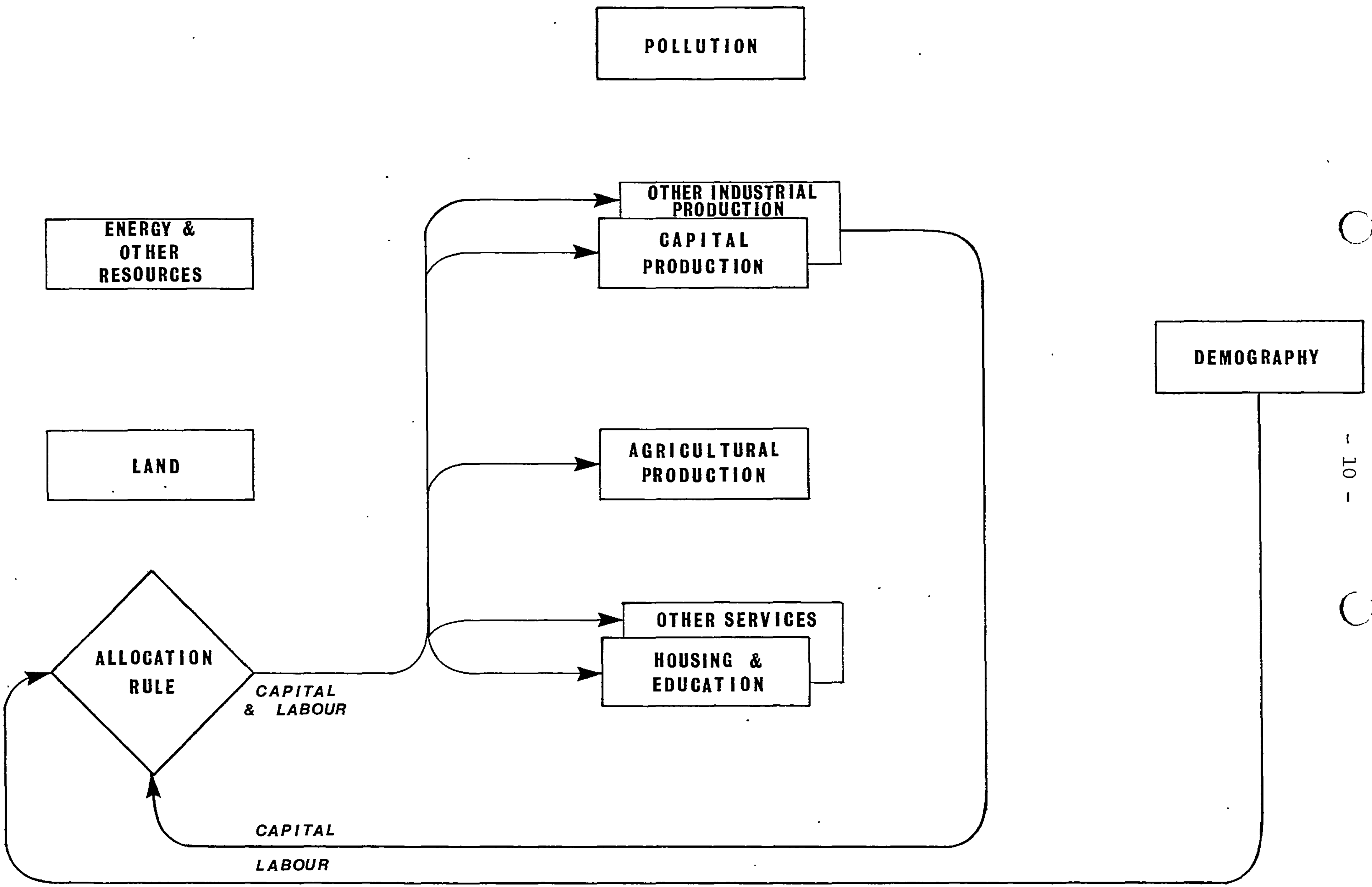

FIGURE 5 : CAPITAL & LABOUR FLOWS IN THE BARILOCHE MODEL

c) Asia including Turkey but excluding Russia, Israel and Lebanon;

d) Africa.

Each of these is assumed to be economically independent from the others in the sense that world trade is neglected by the model. The model does however make provision for large scale economic transfers between blocs in the form of foreign aid.

4. ELEMENTS OF THE LATIN AMERICAN WORLD MODEL

As mentioned in the Overview, a distinctive feature of the Bariloche Model is its normative approach. In terms of the model structure, this creates a need for an optimum-seeking resource allocation mechanism or, in control theory terms, a *controller*. The decisions of the controller are based on an objective function which measures explicitly the desirability of alternative actions taking into account current conditions.

In the Bariloche Model, the basic relationships among the *economic sub-model*, the *demographic sub-model* and the *controller* are shown in Figure 6. The resources to be allocated by the controller are capital and labour for each of the five productive economic sectors. For a particular allocation of capital and labour in one time period, the dynamics of the economic sub-model, based on production functions and capital depreciation, determine the output of goods and services in the next time period. As shown in the figure, the production of goods and services of the *food*, *housing* and *education* sectors is used by the demographic sub-model to determine the labour supply and certain demographic variables.

In what follows, each of the major model components, the economic sub-model, the demographic sub-model and the controller are described separately.

(a) The Economic Sub-Model

Food, housing, education, other consumption and capital goods define productive sectors making up the economic sub-model. The allocation

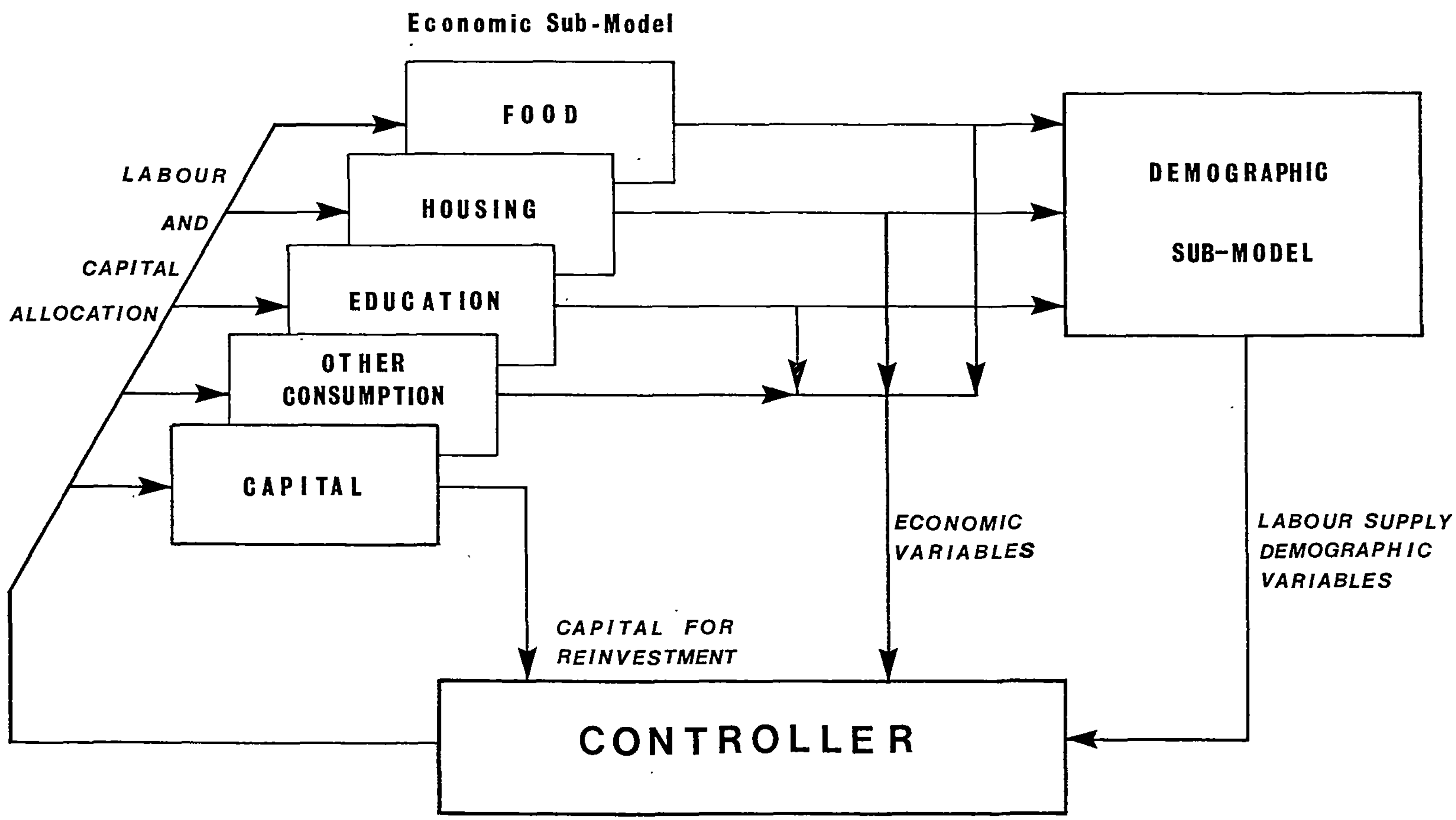

FIGURE 6 ~ INTERACTIONS AMONG ECONOMIC & DEMOGRAPHIC SUB-MODELS AND CONTROLLER rule determines capital and labour inputs to these,* while the output from each sector is determined from a Cobb-Douglas type production function for that sector, as shown in Figure 7. Thus, for sector j with capital input K_j and labour input L_j, the output S_j is given by:

$$S_j = K_j^{\beta_j} L_j^{(1-\beta_j)}$$

where β_j is a parameter for sector j that determines the relative impact of capital and labour on sectoral output. In the Bariloche Model, β_j is assumed constant over time, although provision exists in an expanded version of the model to link these parameters to values of output from the education sector in a way that would allow the production function to respond to increasing levels of education in the work force.

(b) The Demographic Sub-Model

The demographic sub-model transforms the economic variables into various population, life expectancy and other demographic variables. In the Bariloche Model, a general linear transformation structure was assumed for this sub-model. Then historic data from all countries were analysed with numerical regression techniques in order to determine which variables to include and what coefficients to use.

This sub-model is of the form:

$$y = Ax + By + c$$

where the vector y represents the demographic variables to be determined and the vector x represents the given economic variables of the economic

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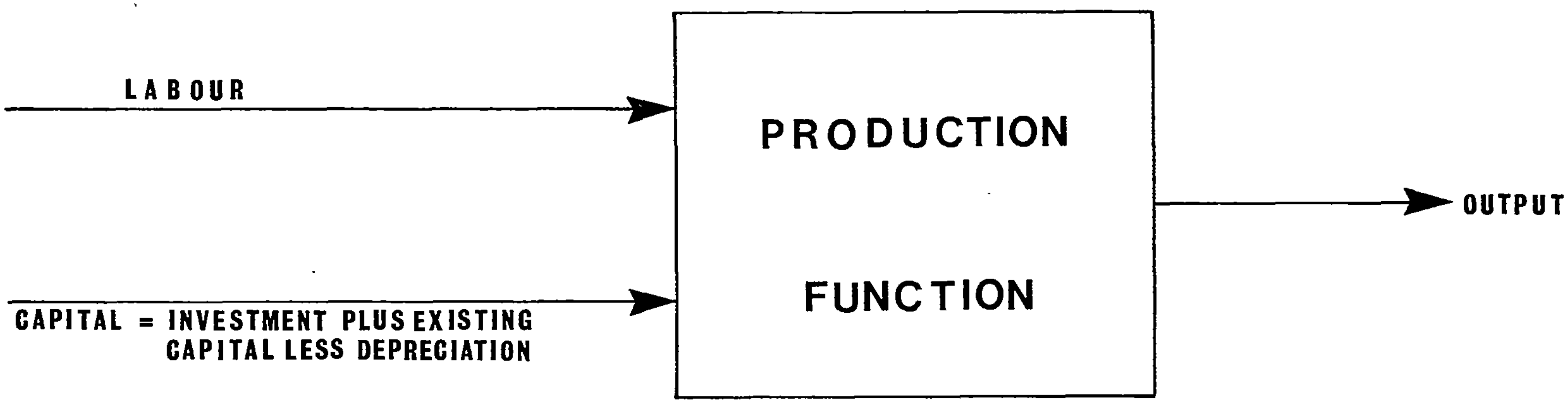
* For a description of how capital and labour are allocated to these sectors, see the section below on the controller.



$$S_{(t)} = K_{(t)}^{\beta_t} \cdot L_{(t)}^{\alpha_t}$$

?

Where $\begin{cases} S = \text{output},\ K = \text{Capital},\ L = \text{labour} \\ \beta_t,\ \alpha_t \text{ are constants},\ \alpha_t = 1 - \beta_t \end{cases}$

FIGURE 7 ~ DETAILS OF A TYPICAL PRODUCTIVE SECTOR sub-model. The matrices A and B and vector c are determined by the regression analysis. By an appropriate transformation of the above equation, the sub-model may be described simply by:

$$y = A'x + c'$$

where A' and c' are calculated from A, B and c above. Appendix I lists the components of the vectors x and y.

(c) The Controller

Operation of the model is split into two phases; first, a projective phase over the period 1960-1980 and then a normative phase from 1980 to 2050. The purpose of the projective phase is to allow the model to assimilate reality and during this phase, the controller does not operate. In 1981, the model enters a normative phase where the controller takes over the task of allocating capital and labour resources.

The controller's function is to assemble all the information about the current state of the system (as described by the model) and choose the best allocation of capital and labour for the next time period. As shown in Figure 6, the controller requires information on the supplies of labour (determined by the demographic sub-model) and capital (determined by the *capital* sector of the economic sub-model) as well as on economic and demographic variables (determined by their respective sub-models). In the Bariloche Model, the objective function which measures the desirability of alternative states of the model is quite complicated in that it balances several conflicting goals on a priority basis. For example, if food production per capita is below a prescribed minimum level, then the food sector receives first priority for labour and capital resources. If food production is at or above its minimum level, priority for resources is given to the education sector. Continuing in a hierarchical manner, if education production achieves its acceptable level, priority reverts to housing and finally, to life expectancy and other consumption which are jointly optimized by maximizing the following product :

(1 + other consumption as a fraction of GNP) X (life expectancy).

As the controller searches for an optimum allocation of labour and capital as measured by this objective function, it must also satisfy a set of additional constraints. These constraints specify that certain variables may not change from year to year by more than a prescribed amount. For example, the fraction of capital in the capital sector is not allowed to decrease and agriculture population must not increase. A listing of these model constraints is given in Appendix II. For a typical year in the optimization phase, the controller searches for an optimum allocation of labour and capital by numerically calculating the value of the objective function for selected feasible allocations. The feasibility of alternative allocations is determined by the set of constraints as mentioned above and once a (computationally) optimal solution is found for the current year, the model variables are updated and the optimization process is repeated for the following year. This is a step by step optimizing approach and is such that the allocation decision for any given year does not depend on possible future states of the system.

5. RESULTS OF THE LATIN AMERICAN WORLD MODEL AND OF THE MEADOWS MODEL

Results of the Latin American World Model indicate that in the developed countries, an acceptable standard of living will be generally achieved in a few years, while in Latin America, this will require some 40 years to attain. In Asia* and Africa, on the other hand, the model predicts a crisis occuring near the millenium. As food shortages develop, the cost of putting new land under cultivation will sap the ability of these countries to meet the basic needs of their populations. This in turn will arrest any further decline in birth rates thereby aggravating the food shortage and economic problems. The model predicts that the economies of these countries will collapse under the strain. This prediction assumes no transfer of economic aid between blocs.

In another run of the model, it was assumed that economic aid in the form of capital would be transferred from the developed countries to Africa and Asia. This aid would begin in 1980 at a level of 0.2% of the GNP rising to a stable value of 2% per year over 10 years. Results of this run indicated that the two blocs could satisfy their populations' basic needs after 65 and 57 years for Africa and Asia respectively.

Table 1 summarizes results of the Meadows Model. Essentially, as the table shows, Meadows predicts world crises arising from resource depletion, food shortage and pollution in roughly that order, but significantly, the model portrays these as inevitable unless population growth can be stabilized.

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* Asia here denotes the country bloc made up of Asia including Turkey but excluding Russia, Israel and Lebanon.

Table 1

Summary of Results from the Meadows Model

POLICY OPTION	RESULT	CRISIS ARISING FROM
Standard	Food per capita peaks at 2006 Population peaks at 2045	Resource depletion
Unlimited resources	Food per capita peaks at 2015 Population peaks at 2133	Pollution
Unlimited resources & pollution controls	Food per capita peaks at 2015 Population peaks at 2162	Food shortage
Unlimited resources Pollution controls Increased agricultural productivity	Food per capita peaks at 2118 Population peaks at 2162	Pollution
Unlimited resources Pollution controls Increased agricultural productivity and perfect birth control	Food per capita peaks at 2124 Population peaks at 2175	Pollution, food shortage, resource depletion
Equilibrium model with stabilized population and conditions of standard model	Food per capita peaks at 2112	-
Equilibrium model with stabilized population and capital	Food per capita peaks at 2118	-

6. CRITIQUE

There are a number of areas in the documentation where our understanding may be limited by the clarity of the report; however, in offering this critique, we have focused on areas where the report is specific.

1. The production functions of the various blocks of countries are constrained to be constant-returns-to-scale functions. Thus, the model assumes that output can always be doubled by doubling the inputs into the production process. This ignores a central problem of world modelling: namely, that the production process may exhibit diminishing returns to scale as the most readily available resources are exhausted. Diminishing returns from some factors of production may be offset by others like increases in technology, but if such an assumption is made, it should be made explicit in the production function. In effect, "technology" should be incorporated into the production function as a variable necessary to maintain the constant returns to scale over time. This would imply that the non-diminishing returns to scale of production result not from virtually unlimited resources but rather from critical improvements in technology. Of course, such a production function would be difficult to justify because it would require evidence that improvements in technology are virtually unlimited. Nevertheless, this is more plausible than assuming constant returns to scale for the world as a whole.

2. The model structure excludes inter-block trade among countries while allocating capital and other productive resources first to agricultural production until prescribed minimum output levels are achieved. If in a particular country, however, productivity is lower in agriculture than in other sectors, then economic development may be retarded. Moreover, it is possible that some blocks of countries will never achieve the minimum standard if their present agricultural productivity is sufficiently low, their population is sufficiently large, and trade is excluded.

Historically, most countries which have experienced economic development have done so by relying heavily on trade and by shifting resources from agriculture to other sectors even if the country was not self-sufficient in food production.

3. As a consequence of its normative approach, the Bariloche Model incorporates a controller for allocating capital and labour resources in a way that optimizes an objective function. This optimization process is performed year by year and not over the entire time period of the model.

An important consideration with respect to this method of optimization is that drastically different solutions result when the <u>entire</u> planning period is considered. It is a general characteristic of dynamic capital allocation problems that early in the planning period consumption must be foregone in order to build up the supply of capital so that future consumption can be greater. Myopic* solutions usually eschew this policy and direct the bulk of investment toward consumption. This tendency has

* Myopic here is used in the technical sense of taking into account only current conditions.

been avoided in the Bariloche Model, artificially, by constraining the minimum level of capital allowed for the capital sector.

In Appendix III, a simple example of a dynamic investment problem is formulated and analysed to show the general nature of the optimal investment policies over time. The solution in this simple case is to put all investment into capital production up to a certain time and then to invest in consumption afterwards. This solution minimizes the time to reach a certain amount of production or equivalently, maximizes the amount of production for a given planning period. Also in the appendix, a simplified version of the world model is formulated to show the relevance of the simple example.

It is clear that dynamic optimization over the entire planning period is preferable to year by year optimization, and that potential improvements are sufficient to warrant a thorough investigation of the feasibility of this approach.

4. There are a number of points to be made in connection with the demographic sub-model. These are: first that it is not reasonable to assume that, observed relationships in recent data will be valid throughout the time period under study, second that as a projective model, the demographic sub-model is not well specified,* and third that for statistical reasons, the sub-model may be invalid. In what follows, each of these is treated separately.

a) The data used to develop the model is for the period 1960-1970 and for all countries for which data is available. Since countries do not change very much in a ten year period but differ

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* Specified is used here in the technical sense of identifying model variables and their assumed interrelationships.

enormously among themselves, these data can be considered as cross sectional. There is implicit, then, an assumption that the time development of individual countries will follow a similar path to that observed between developed and underdeveloped countries at the present. Since the model is intended to describe a path of economic development that is radically different from what would transpire if historical trends were extrapolated, this use of this data is inconsistent with the normative modelling approach.

b) Valid empirical modelling includes as part of the procedure a critical evaluation of the equations obtained to ensure that they are reasonable and that unnecessary terms have not been retained. Specific indications that specifications of the model may be inadequate are as follows:

i) Several variables enter the right-hand side of some equations with the theoretically incorrect sign.*

ii) Dimensions of many of the equations are confused as a result of stocks and flows being arbitrarily mixed together as explanatory variables in most equations.

iii) Equations used in the demographic sub-model are linear. Since the development of population in time is known to be highly non linear, extrapolation with these equations beyond the data base may not be justified.

c) Discussions in the report fitting the model deal with the requirement of stable numerical techniques because of high correlations

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* For example, all three equations which define population variables indicate that population increases as food consumption decreases. See Appendix I for a complete listing of the equations of this sub-model.

among the independent variables. The statistical implication of this is that there is a high degree of multicolinearity in the data. When multicolinearity is high, there are no unique values of the parameters which fit the data; instead, there is an infinity of possible values all of which will do almost as good a job in predicting the existing set of data. Thus, it is doubtful that these equations will predict adequately.

APPENDIX I

EQUATIONS OF DEMOGRAPHIC SUB-MODEL

The demographic sub-model is of the form:

$$y = Ax + By + c$$

where y is a vector of demographic variables, x is a vector of economic variables (plus other demographic variables defined elsewhere in the Bariloche Model), A and B are matrices and c is a vector, all derived by regression. The components of the vectors x and y are listed in Table 1 along with the Bariloche Model acronyms. The complete set of equations is listed below.

$$LE = 0.057\ PR - 0.15\ AGP + 0.25\ SEP + 0.27\ EN + 0.013\ HR + 39.135$$

$$CHM = -0.16\ PR + 1.06\ AGP - 0.275\ SEP - 0.92\ EN + 118.94$$

$$BHR = -0.74.10^{-3} CC - 0.05\ PR - 0.11\ SEP - 0.06\ EN - 0.32\ LE - 0.65\ HR + 72.57$$

$$GRM = -0.64.10^{-3} CC - 0.003\ PR - 0.075\ BHR - 0.31\ LE + 0.04\ AGP + 0.04\ CHM + 30.368$$

$$PF = 0.023\ BHR - 0.08\ GRM - 0.001\ CHM - 0.009\ UR - 0.06\ HR + 5.976$$

$$P09 = 0.37\ BHR + 0.09\ LE - 0.23\ GRM - 0.04\ PR - 0.42\ SEP + 17.715$$

$$P1014 = -0.2.10^{-3} TC - 0.3610^{-3} CC + 0.07\ BHR + 0.02\ LE - 0.05\ GRM - 0.13\ SEP + 10.29$$

$$P1519 = -0.59.10^{-3} TC - 0.42.10^{-3} CC + 0.02\ BHR - 0.02\ GRM - 0.1\ SEP - 0.01\ HR + 12.06$$

These equations may be solved for y in terms of x only to yield the following form:

$$y = A'x + c'$$

where A' and c' are derived from A, B and c above. The resulting components of x that appear in each equation for the y variables are as listed in Table 2.

Table 1

Demographic Sub-Model Variables

Input Variable Component of x	Acronym	Definition
1	PR	Proteins
2	AGP	Agriculture population
3	SEP	Secondary population
4	EN	Enrollment
5	HR	House rate
6	CC	Calories from cereals & starchy foods
7	UR	Urbanization
8	TC	Urban population

Output Variable Component of y	Acronym	Definition
1	LE	Life expectancy
2	CHM	Child mortality
3	BHR	Birth rate
4	GRM	Gross mortality
5	PF	Persons per family
6	P09	Population age 0-9
7	P1014	Population age 10-14
8	P1519	Population age 15-19

Table 2

Interrelationships of Demographic Sub-Model

Component of y	Definition	Components of x involved
1	Life expectancy	1 to 5
2	Child mortality	1 to 4
3	Birth rate	1 to 6
4	Gross mortality	1 to 6
5	Persons per family	1 to 7
6	Population age 0-9	1 to 6
7	Population age 10-14	1 to 6 and 8
8	Population age 15-19	1 to 6 and 8

APPENDIX II

CONSTRAINTS INCORPORATED IN THE BARILOCHE MODEL

The following is a listing of some of the constraints that are part of the Bariloche Model. Some "constraints" that are listed in the model documentation are really components of the objective function and are, therefore, not listed here.

1. The fraction of investment in the capital sector at any time must not be less than 0.5% below the value at the beginning of the optimization phase.
2. Agricultural population must not increase over time.
3. All prices are positive.
4. The fraction of the labour force employed in the housing sector must not be greater than 0.8%.
5. The price of education services must not vary by more than 4% from its previous year's value.
6. The fraction of consumption to GNP at any time must not be less than 42%.
7. Enrollment in education must not increase by more than 10% any year.
8. The quantity of calories per person must not decrease.
9. The number of housing units per family must not decrease.
10. Enrollment in education must not decrease.
11. The price of housing should not increase as long as there is adequate food production.

12. Life expectancy must not decrease.
13. The fraction of investment in the capital sector to total GNP is constrained to lie between an upper and a lower limit.
14. The distribution of labour resources from sector to sector must not change by more than 2% per year.
15. The distribution of capital resources from sector to sector must not change by more than 6% per year.
16. Prices cannot change by more than 1% per year.

APPENDIX III

PRODUCTION PLANNING: MYOPIC VERSUS FAR-SIGHTED THE FARMER'S ALLOCATION PROBLEM

The implications of short range planning versus long range planning are demonstrated in this appendix by a simple allocation problem in agriculture production. A highly simplified but representative version of the Bariloche Model is then developed to show the relevance of the Farmer's Allocation Problem* to the world model. Hence some insight into the implications of the choice of year-by-year rather than long range optimization of the Bariloche Model is obtained.

The Farmer's Allocation Problem

In this model, a farmer wishes to produce a single crop over a long fixed time period so as to maximize the total amount in storage at the end of the time period. He must decide how much of his crop to store and how much to sell and reinvest each year in order to achieve his objective.

By definition, we have:

$x(t)$ = rate of production at time t

$u(t)$ = fraction of production rate that is reinvested at time t

and by assumption:

$$x(t) = u(t)x(t) \quad x(0) > 0. \tag{1}$$

The objective of the farmer is represented by the maximization of:

$$\int_0^T (1 - u(t))\, x(t)dt \tag{2}$$

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* Adapted from D.G. Luenberger, Optimization by Vector Space Methods, New York: Wiley, 1969.

with respect to u where

$$0 \leq u(t) \leq 1 \tag{3}$$

and T is the end of the planning period.

The optimal reinvestment policy is easily determined by the Pontryagin Maximum Principle. The Hamiltonian for this problem is:

$$H(x,u,\lambda) = \lambda(t)u(t)x(t) + (1 - u(t))x(t) \tag{4}$$

where $\lambda(t)$ satisfies the adjoint equation

$$-\lambda(t) = u(t)\lambda(t) + 1 - u(t) \qquad \lambda(T) = 0 \tag{5}$$

The Maximum Principle states that an optimal solution must maximize the Hamiltonian with respect to all admissible control inputs u. A rearrangement of (4) gives:

$$H(x,u,\lambda) = u(t)(\lambda(t) - 1)x(t) + x(t) \tag{6}$$

and, since $x(t)$ is positive, the Hamiltonian is maximized by:

$$u(t) = \begin{cases} 1 & \lambda(t) > 1 \\ 0 & \lambda(t) < 1 \end{cases}$$

Now (5) can be solved to yield the optimal solution, which is that the farmer should reinvest all production ($u(t) = 1$) up to time T-1 and then store all production ($u(t) = 0$).

This sample problem also demonstrates that the same investment policy would minimize the time to reach a given level of production. That is, if the farmer's objective was to minimize to the time T to fill his silo, then the optimal investment policy would be to reinvest everything until the last year of production.

A Simplified World Model

The relevance of the Farmer's Allocation Problem to the Bariloche Model may be more apparent if a very simple world model of production is formulated in the Bariloche fashion.

In the simple model, suppose there are two productive economic sectors: capital and other production. That is, assume that all but the capital sector of the Bariloche Model are collapsed into one sector. Then the production rates $S_1(t)$ and $S_2(t)$ of these two sectors are given by their respective production functions:

$$
\begin{aligned}
S_1(t) &= K_1(t)^{\alpha_1} L_1(t)^{1-\alpha_1} \\
S_2(t) &= K_2(t)^{\alpha_2} L_2(t)^{1-\alpha_2}
\end{aligned}
\tag{7}
$$

The controller of this simple model must allocate investment produced by the capital sector (sector 2 above) to the two sectors over time in a way which minimizes the time required to reach a particular level of production.

This model can be reformulated slightly to emphasize the dependence on this capital allocation. First, we assume that the labour supply is constant over time so that the variables $L_1(t)$ and $L_2(t)$ can be replaced by a constant. Then two equations representing the capital flows in the two sectors can be written. These are:

$$
\begin{aligned}
\dot{K}_1 &= a_2 u(t) K_2(t)^{\alpha_2} - d_1 K_1(t) \\
\dot{K}_2 &= a_2 (1 - u(t)) K_2(t)^{\alpha_2} - d_2 K_2(t)
\end{aligned}
\tag{8}
$$